The Game of 31 and the Euclidean Algorithm

Task 1
In this task, you will play the following game:
1. Each student has the numbers 1, 2, 3, 4, 5, 6, which can be represented with the cards A, 2, 3, 4, 5, 6.
2. One student starts by choosing a number.
3. The second student continues by choosing a number, which is added onto the number from student 1.
4. This process continues until one student reaches 31. Note that all of the numbers can be repeated.

After playing this game several times, you may begin to notice a pattern. In a clearly written paragraph, answer each of the following questions:

i. If you are the first student, what number should you choose to win the game?

ii. Are there certain numbers you need to reach in order to ensure that you will win?

iii. Is there a general strategy for the game that allows you to win each time?

Task 2
You can repeat task 1 by choosing numbers other than 31, and allowing the highest card to be different from 6. For instance, you can play the game of 40 with the numbers 1, 2, 3, 4, 5, 6, 7 or the game of 75 with 1, 2, 3, 4.

Is there a strategy that allows you to win the game of \(x\) if you use card numbers 1, 2, 3, 4......\(k\)? Explain your answer in a clear paragraph.
Research Task
The game that you played earlier is based on a mathematical division technique called the 'Euclidean Algorithm', named after the famous mathematician Euklid from Alexandria.

By using appropriate resources, do research about Euclid and the Euclidean Algorithm. What is an 'algorithm'? How does Euclid's algorithm for division work and how can it help you win the game of 31 (or the game of x in general)?