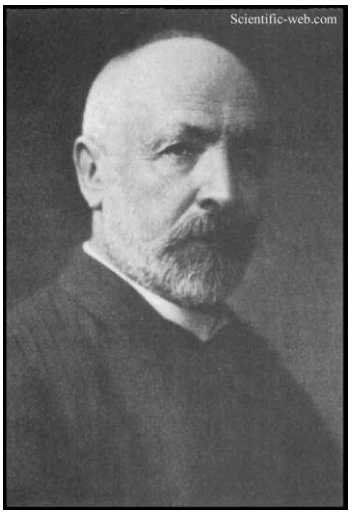
Math Project - Number Bases

In elementary and middle school mathematics, students are introduced to place value. We often write numbers in base 10, meaning that each digit represents a power of 10. Notice that the coefficients (the numbers before each power of 10) are all between and 0 and 9. Why is this?

Example:

$$243560.02 = 2*10^5 + 4*10^4 + 3*10^3 + 5*10^2 + 6*10^1 + 0*10^0 + 0*10^{-1} + 2*10^{-2}$$

In many applications, it is important to write numbers using a different base. For example, in computer science, base 2 (binary) and base 16 (hexadecimal) are used in order to represent numbers that correspond to switches being turned on or off. In pure mathematics, other bases can also be used to examine important mathematical properties. An example would be the Cantor fractal, which uses numbers in base 3, first examined by the German mathematician Georg Cantor.



Writing Numbers in Binary

In the binary number system, all numbers are written using powers of 2. Notice that the coefficients are all 0's and 1's.

Example: The number 100110 is equal to $1*2^5 + 1*2^2 + 1*2^1$=38
On the other hand , the number 140 is equal to 128 +8 +4
Which can be written as $2^7 + 2^3 + 2^2$ =10001100

Task

1. Write the numbers 61, 73, 212 and 1000 in binary.
2. What can you say about a number that ends in 1 when you write it in binary? What about a number that ends in a 0?
3. What can you say about a number whose binary representation contains a 1, followed by all zeros?

The Hexadecimal System

The hexadecimal system is a base 16 system that is often used in computer science in order to help deal with binary strings that are of length 8 (bytes). The reason this is done is that a binary number with eight digits has a term that is equal to 2^8=256. Since 16^2 =256, bytes can be represented using two digits in the hexadecimal system, making it easier to work with algebraically.

The hexadecimal system contains the numbers 0-9 and a,b,c,d,e and f for the numbers 10,11,12,13,14,15.

Example: 240678 = $3*16^4 + 10*16^3 + 12*16^2 + 2*16 + 6$
=3bd26

Task

By using appropriate resources, write a paragraph about the hexadecimal system, outlining its development and use in computer science. Convert the number 254367839 into hexadecimal. How would you convert the number 11111111111 from hexadecimal into base 10?

Number Bases and Geometric Series

Fill in the table below:

Number in Base 10	Number in Base 2	Number in Base 3	Number in Base 4	Number in Base 5
1				
11				
111				
1111				
11111				
111111				

The goal of this task is to find out how to calculate a 'k' digit number in base 'b' consisting entirely of 1's.

Using appropriate resources, find out information about 'geometric sequences' and series. In particular, answer the following questions?

1. What is a geometric sequence and how is it defined?

2. How do you calculate the sum of the first k terms of a geometric sequence with common ratio b?
3. Explain how the number 11111….1111 in base b forms a geometric series, and demonstrate how you could convert 11111111111 in base b to base 10.
4. Extend your pattern from question 3 to numbers such as 2222222 or 3333333. In general, how do you convert nnnnnnnnnnnn from base b into base 10, if n<b?