

Grade 8 Math Project - Fermat's Square Theorem

Since the time of the ancient Greeks, mathematicians have been fascinated by the strange properties of numbers. They knew that all whole numbers could be broken up into the product of **prime** numbers. For example, $72 = 2^3 3^2$. Prime numbers themselves are also very interesting mathematical structures that are the focus of extensive research currently. In this assignment, you will classify prime numbers into one of three groups, and you will investigate an interesting property that primes belonging to one of the groups possess. This is the so called 'Fermat Square' property, attributed to the French mathematician Pierre de Fermat (see background).

Task

Answer each of the following questions. Make sure to give complete explanations for your work.

1. How many even prime numbers are there?
2. List all of the prime numbers between 1 and 50. Divide each of these numbers by 4. What is the remainder you get?
3. Fermat stated that every odd prime number p fits into 1 of two groups, namely $p = 4n + 1$ or $p = 4n + 3$. For the prime numbers between 1 and 50, place them in the appropriate place in the table below:

$P = 4n + 1$	$P = 4n + 3$

4. The numbers 1, 4, 9, 16, 25, 36, 49, are called square numbers because they are the areas of squares with side length 1, 2, 3, 4, 5, 6, 7,

Which of your prime numbers can be written as the sum of two square numbers?